Role of gluons and the quark sea in the proton spin *

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The real, interacting elementary particle always consits of a 'bare' particle and a cloud of virtual particles mediating a self-interaction and/or the bond inside a composite object. In this letter we discuss the question of spin content of the virtual cloud in two different cases: electron and quark. Further, the quark spin is discussed in the context of proton spin, which is generated by the interplay of quarks and virtual gluons. We present a general constraint on the gluon contribution and make a comparison with the experimental data.

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1. INTRODUCTION

In our recent paper [1] we studied the proton spin structure in leading order of the covariant approach assuming the gluon contribution to the proton spin can be neglected. However the question of the real role of gluons in generating the proton spin is still open. Actually some recent data obtained at RHIC and their analyses [2–6] can suggest a positive gluon contribution to the proton spin. The main aim of the present letter is to extend discussion from our previous study to the case of nonzero gluon contribution. We will show what constraint on the gluon contribution follows from the covariant approach.

Sec. 2 is devoted to a discussion about some general aspects of particle spin and its scale dependence. Two different examples are considered, electron and quark. The electron spin structure is also an interesting topic, see the recent study [7] and the previous papers [8, 9]. Particularly important questions concern the spin of quarks inside the nucleon. In Sec. 3 the discussion about the proton spin, which is generated by the interplay of angular moments of quarks and gluons, continues in the context of recent experimental data.

In present calculations we use, as before, the rest frame of the composite system as a starting reference frame. This frame is suitable for the consistent composition of spins and OAMs of the constituents in the representation of spinor spherical harmonics. The resulting state serves as an input for construction of the covariant quantities, like the spin vector or the spin structure functions [1].

2. SPIN OF THE PARTICLE IN ITS SCALE DEPENDENT PICTURE

In general, description of real interacting particles can be related to their 'bare' or 'dressed' form. In our present discussion we address the general questions:

a) How much do the virtual particles surrounding bare particle contribute to the spin of corresponding real, dressed particle?

b) How much do the virtual particles mediating bond of the constituents of a composite particle contribute to its spin?

In quantum mechanic the total angular momentum (AM) of any particle including composite ones is given by the sum of the orbital AM (OAM) and spin, $\mathbf{J} = \mathbf{L} + \mathbf{S}$. The corresponding quantum numbers are discrete sets of integers or half-integers and in the relativistic case only total AM conserves, so only J and J_z can be the good quantum numbers. We will illustrate the problem with two different examples, electron and quark.

2.1. Spin of electron

The electron, as a Dirac particle, in its rest frame has AM defined by its spin, $s = 1/2$. This value is the same for the dressed electron (as proved experimentally) and for the bare one (as defined by the QED Lagrangian). The

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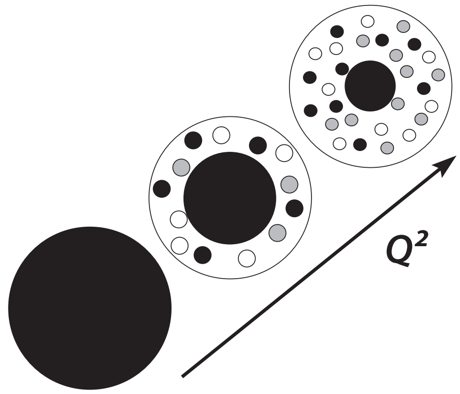

FIG. 1: Scale dependent image of a real particle, see text.

dressed electron is a bare electron surrounded by the virtual cloud of γ and e^-e^+ pairs, as symbolically sketched in Fig. 1 for different scales represented by the parameter Q^2. So the renormalization as a continuous change of the scale should not change the AM represented by the discrete numbers, $J^e(Q^2) = s = 1/2$. But what about the projections $J_z^e(Q^2)+ J_z^\gamma(Q^2) = \pm 1/2$? Can the contribution of virtual cloud $J_z^\gamma(Q^2)$ differ from zero and how much? In this letter we present a semiclassical estimate of the vector $\mathbf{J}^\gamma$. The electromagnetic field, or its γ-quanta, are according to Maxwell equations created by the electric current. We consider the current generated by the electron states represented by the spinor spherical harmonics

$$|j, j_z\rangle = \Phi_{jl_pj_z}(\mathbf{r}) = \frac{1}{\sqrt{2\epsilon}} \begin{pmatrix} \sqrt{\epsilon+m} R_{kl_p} \Omega_{jl_pj_z}(\omega) \\ -\sqrt{\epsilon-m} R_{k\lambda_p} \Omega_{j\lambda_pj_z}(\omega) \end{pmatrix}, \tag{1}$$

where ω represents the polar and azimuthal angles (θ, φ) of the space coordinates $\mathbf{r}$ with respect to the axis of quantization z, $l_p = j \pm 1/2,\ \lambda_p = 2j - l_p$ (l_p defines the parity), energy $\epsilon = \sqrt{\mathbf{k}^2 + m^2}$ and

$$\Omega_{jl_pj_z}(\omega) = \begin{pmatrix} \sqrt{\frac{j+j_z}{2j}} Y_{l_p, j_z-1/2}(\omega) \\ \sqrt{\frac{j-j_z}{2j}} Y_{l_p, j_z+1/2}(\omega) \end{pmatrix}, \tag{2}$$

$$\Omega_{j\lambda_pj_z}(\omega) = \begin{pmatrix} -\sqrt{\frac{j-j_z+1}{2j+2}} Y_{\lambda_p, j_z-1/2}(\omega) \\ \sqrt{\frac{j+j_z+1}{2j+2}} Y_{\lambda_p, j_z+1/2}(\omega) \end{pmatrix},$$

where $l_p = j - 1/2$ and $\lambda_p = j + 1/2$. The functions Y_{l,l_z} are usual spherical harmonics. The radial functions in the case of free electron read:

$$R_{kl}(r) = \sqrt{\frac{2\pi k}{r}} J_{l+1/2}(kr), \tag{3}$$

$$\int r^2 R_{kl} R_{k'l} dr = 2\pi\delta(k - k'),$$

where $k = |\mathbf{k}|$ and $J_\nu(z)$ are Bessel functions of the first kind, otherwise (e.g. for electron in the hydrogen atom) the radial functions differ according to an external field. However, it is important that the information about the electron AM is completely absorbed in the angular terms and does not depend on the radial functions. The states $|j, j_z\rangle$ are eigenstates of the total AM and have been discussed before [1] in momentum representation, while in the present note we deal with their coordinate representation corresponding to the rest frame of an composite system. The corresponding current reads

$$I_\mu = (I_0, \mathbf{I}) = \Phi^\dagger_{jl_pj_z}(\mathbf{r}) \gamma^0 \gamma_\mu \Phi_{jl_pj_z}(\mathbf{r}) \tag{4}$$

and one can check that

$$I_0 = h_I \rho_{j,j_z}(\cos\theta), \qquad \mathbf{I} = h_{II} \rho_{j,j_z}(\cos\theta)\, \mathbf{r}, \tag{5}$$

j, j_z	$\rho_{j,j_z}(\omega)$
$\frac{1}{2}, \pm\frac{1}{2}$	1
$\frac{3}{2}, \pm\frac{3}{2}$	$\frac{3-3\cos 2\theta}{4}$
$\frac{3}{2}, \pm\frac{1}{2}$	$\frac{5+3\cos 2\theta}{4}$
$\frac{5}{2}, \pm\frac{5}{2}$	$\frac{45-60\cos 2\theta+15\cos 4\theta}{64}$
$\frac{5}{2}, \pm\frac{3}{2}$	$\frac{57-12\cos 2\theta-45\cos 4\theta}{64}$
$\frac{5}{2}, \pm\frac{1}{2}$	$\frac{45+36\cos 2\theta+15\cos 4\theta}{32}$

TABLE I: The examples of the angular distributions ρ_{j,j_z}. The common factor $1/4\pi$ is omitted.

where

$$\begin{aligned} h_I &= \frac{1}{2}\left(\left(1+\frac{m}{\epsilon}\right)R_{kl_p}^2 + \left(1-\frac{m}{\epsilon}\right)R_{k\lambda_p}^2\right), \\ h_{II} &= -\frac{k}{\epsilon r}R_{kl_p}R_{k\lambda_p}. \end{aligned} \tag{6}$$

A few examples of ρ_{j,j_z} are given in Table I, where one can observe the following. The stationary current I_μ depends only on j and $|j_z|$, therefore it does not involve any information on the direction of electron polarization. So, there is no reason to expect any correlation between electron polarization and polarization of the electromagnetic field generated by this current, or equivalently polarization of the statistical set of emitted and reabsorbed γ. In other words the average polarization of virtual cloud of γ and consequently also e^-e^+ pairs should be zero. The AM of the electromagnetic field is given by the relation

$$\mathbf{J}^\gamma = \int \mathbf{r} \times (\mathbf{E} \times \mathbf{H}) d^3\mathbf{r}, \tag{7}$$

where $\mathbf{E}, \mathbf{H}$ are the corresponding intensities of electric and magnetic field. Due to the symmetry of current (5) that generates these fields, the corresponding AM satisfies

$$\mathbf{J}^\gamma = 0, \tag{8}$$

the proof is given in Appendix. This relation follows only from the angular terms in the wave function (1) and does not depend on the radial ones. In other words, the relation (8) holds not only for a free electron, but also for an electron bound in the hydrogen atom. The result represents a mean value, which is not influenced by the fluctuations generated by single γ. So, this calculation suggests the integral AM of the cloud of virtual γ is zero despite the fact that AM of its source, the electron in a state (1), is not zero. While the free electron emits and reabsorbs virtual photons by itself, the electron bounded in hydrogen atom in addition exchanges (emits and absorbs) virtual photons with the proton. Since the AM of the electromagnetic field generated by the proton is zero as well, the total AM of hydrogen will be given only by AMs of the electron and proton, without contribution of the electromagnetic field generated by both the particles.

Similar arguments can be relevant also for atoms in general and perhaps for the nucleons bound in a nucleus. This would suggest the virtual particles mediating the binding of nucleons also do not contribute to the resulting spin of nucleus, which must be always integer or half-integer.

Our approach (A) has a common basis with the QED calculation (B) suggested in Ref. [7], since both the approaches follow from the general QED equations (1),(6) and (7) in the last reference. Despite that, there are some differences between them, the most apparent are as follows.

(A) The approach is semiclassical only and the AMs are directly related to the electron wavefunction ψ and the classical electromagnetic field A_μ generated by the electric current $\bar{\psi}\gamma_\mu\psi$. The preferred reference frame is the frame defined by the spinor spherical harmonics (1) or the rest frame of the defined composite system (e.g. atom). This simplified treatment allows us to obtain the relation (8) but without a decomposition into the spin and OAM parts.

(B) The study is focused on the fundamental problem of the AM decomposition in quantum field theory and explicit calculation is performed for the QED. The preferred reference frame is the infinite momentum frame. The light-front formalism is adopted to achieve a compatibility with the standard formulation of the parton model. The result on the total boson (photon) AM seems be also rather small, $S_b + L_b \simeq \mathcal{O}(e^2)$.

So both the approaches, adopting rather different formalism and correspondingly also some different assumptions, are not contradictory. From a phenomenological point of view, they are complementary.

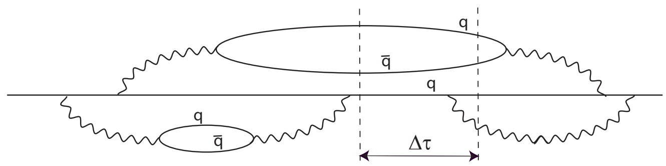

FIG. 2: Scale dependent image of a valence quark, see text.

2.2. Spin of quark

The situation with quarks inside a nucleon is more complicated. The quark at different scales is sketched in Fig. 1 (in which the bare electron surrounded by virtual cloud of γ and e^-e^+ pairs is now replaced by the bare quark with a cloud of virtual g and $q\bar{q}$ pairs). The terminology is as follows:

i) The bare quark can be identified with the current quark, which can be described by the distribution functions $q^a(x)$ defined in the quark-parton model. They are related to the sets of quarks and antiquarks in the figure for $Q^2 \to \infty$.

ii) The constituent quark can be identified with the dressed quark at a low Q^2 scale.

iii) The valence quark can be identified with the set of quarks, from which the cloud of virtual $q\bar{q}$ pairs and gluons is separated off. In the figure the valence quarks are represented by the central spots. Strictly speaking, depending on the scale, valence and sea quarks may not be clearly distinguishable. In a short time interval $\Delta\tau$, a quark from the virtual $q\bar{q}$ pair is indistinguishable from the source, valence quark, see Fig. 2. However, the usual definition in terms of the quark-parton model distributions

$$q^a_{val}(x,Q^2) = q^a(x,Q^2) - \bar{q}^a(x,Q^2) \tag{9}$$

is unambiguous. For quarks the parameter Q^2 represents the renormalization scale, but also the DIS parameter ($-Q^2$ = photon four-momentum square) or equivalently a scale of the space-time domain inside which the photon absorption takes place [1].

Now, we can put the question b) for the quarks bound inside the proton: How much the field of virtual gluons generated by the valence quarks and the sea of virtual $q\bar{q}$ pairs created by the gluons contribute to the proton spin? This question is being studied in the experiments, which measure contribution of the gluons and sea quarks to the proton spin. Available data from the experiments COMPASS [10] and HERMES [11] suggest rather small gluon contribution, in fact the data are consistent with zero within statistical errors. The very recent results of the RHIC experiments [2–6] suggest a positive gluon contribution which, however still cannot fully compensate for a small quark contribution to the proton spin.

These results can be interpreted in the framework of covariant approach presented in Ref. [1]. In the paper we studied the relativistic interplay between the quark spins and OAMs, which collectively contribute to the proton spin. The simplest scenario assuming

1) the quarks are in the state $j = 1/2$, see Eq. 113[1],

2) mass of quarks can be neglected, $\langle m/\epsilon \rangle \to 0$,

3) there is no gluon contribution, i.e. proton spin $J = 1/2$ is generated only by the AM of quarks, see Eq. 109[1] gave a prediction for the contribution of the quark spins in DIS region,

$$\Delta\Sigma = \frac{1}{3}, \tag{10}$$

while the “missing” part of the proton spin is fully compensated by the quark OAM. This prediction fits the data [12–14] surprisingly well.

However, in a more general case, if only condition 1) is assumed, then AM of each quark consists of the spin and OAM part

$$\langle s_z \rangle = \frac{1+2\tilde{\mu}}{3} j_z, \quad \langle l_z \rangle = \frac{2-2\tilde{\mu}}{3} j_z, \quad \frac{\langle l_z \rangle}{\langle s_z \rangle} = \frac{2-2\tilde{\mu}}{1+2\tilde{\mu}}, \tag{11}$$

where $j_z = \pm 1/2$, see Eqs. (17), (22) in Ref. [1]. It is very important result, which easily follows from the algebra of spinor spherical harmonics representing the solutions of Dirac equation. The ratio $\tilde{\mu} = \langle m/\epsilon \rangle$ here plays a crucial role, since it controls a ”contraction” of the spin component, which is compensated by the OAM. It is a quantum

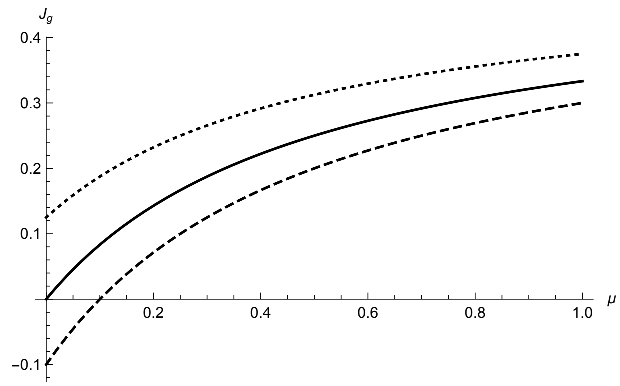

FIG. 3: Dependence of $\Delta\Sigma$ on J_g and $\tilde{\mu}$. The dotted, full and dashed lines correspond to $\Delta\Sigma = 0.25, 0.33$ and 0.4 respectively.

mechanical effect of relativistic kinematics. Mathematically, a small $\tilde{\mu}$ means the lower component of Dirac spinor is important. This is also the case in [15, 16], where with some distinction of formalism and paradigm the authors come to similar results on the spin of quark bound in the proton. The effect is strongly correlated with the transversal motion of quarks inside the nucleon [17].

The relations (11) imply that the total AM of a composite system of quarks with $j_1 = j_2 = j_3 = ... = 1/2$ reads

$$J^q = \langle \mathbb{S}_z \rangle + \langle \mathbb{L}_z \rangle \,, \tag{12}$$

where the ratio of the total spin $\langle \mathbb{S}_z \rangle$ and OAM $\langle \mathbb{L}_z \rangle$ is the same as for the one-quark states above:

$$\frac{\langle \mathbb{L}_z \rangle}{\langle \mathbb{S}_z \rangle} = \frac{2 - 2\tilde{\mu}}{1 + 2\tilde{\mu}}. \tag{13}$$

Further, if the proton spin consists of the quark and gluon contributions, one can write

$$\frac{1}{2} = J^q + J^g; \qquad J^q = \frac{1}{2}\varkappa, \qquad J^g = \frac{1}{2}\left(1 - \varkappa\right). \tag{14}$$

With the use of Eqs.(12)-(14) one gets

$$\begin{gathered} \langle \mathbb{S}_z \rangle + \langle \mathbb{L}_z \rangle = \langle \mathbb{S}_z \rangle \left(1 + \frac{2 - 2\tilde{\mu}}{1 + 2\tilde{\mu}}\right) = \frac{1}{2}\varkappa, \\ \varkappa = 1 - 2J^g, \end{gathered} \tag{15}$$

which after replacing $\langle \mathbb{S}_z \rangle = \Delta\Sigma/2$ gives

$$\Delta\Sigma = \frac{1}{3}\left(1 - 2J^g\right)\left(1 + 2\tilde{\mu}\right). \tag{16}$$

In a higher approximation, if we admit an admixture of the quark states with $j \geq 3/2$, the relation is modified as

$$\Delta\Sigma \lesssim \frac{1}{3}\left(1 - 2J^g\right)\left(1 + 2\tilde{\mu}\right). \tag{17}$$

3. DISCUSSION AND CONCLUSION

The relation (16) means the quark spin content depends on two parameters, the gluon contribution J^g and the quark effective mass ratio $\tilde{\mu}$, which affects proportion of the quark OAM. It follows from the algebra of spinor spherical harmonics and from general rules of AM composition in the system of quarks and gluons with the total spin $J = 1/2$. The dependence is demonstrated in Fig. 3. One can observe:

a) $\Delta\Sigma \leq 1/3$ corresponds to $J^g \geq 0$ for any $1 \geq \tilde{\mu} \geq 0$. A special case $\Delta\Sigma = 1/3$ and $\tilde{\mu} \to 0$ implies $J^g \to 0$.

b) $\Delta\Sigma > 1/3$, then the sign of J^g depends on $\tilde{\mu}$. Apparently, $J^g < 0$ would imply $J^q > 1/2$.

In this way the COMPASS and HERMES data [12–14] giving $\Delta\Sigma \approx 1/3$, can be compatible also with a positive gluon contribution J^g suggested by the recent data on RHIC. Note that positive J^g correlates with a positive quark

effective mass ratio $\tilde{\mu}$. Can we somehow estimate this parameter? In the proton rest frame a quark has momentum $\langle k\rangle = \sqrt{3/2}\,\langle k_T\rangle$ and there are independent ways to estimate it, for example:

1) If we take proton diameter $d_p = 0.84$fm, then the uncertainty relation gives for the corresponding momentum roughly $k \approx 230$MeV.

2) The analysis [18–21] of the data on the azimuthal asymmetry in semi-inclusive DIS suggest $\langle k_T\rangle \approx 400-600$MeV.

3) The statistical approaches [22–24] suggests $\langle k\rangle \approx 40-100$MeV and a similar value was obtained in Ref. [25] for valence quarks, see also discussion in Ref. [26, 27].

Therefore, if one suppose the quark effective mass of the order MeV, then the parameter $\tilde{\mu} \lesssim 0.1$, which gives a similar upper limit on the gluon contribution, $J^g \lesssim 0.1$. Finally, the spin contribution of the sea quarks is known to be small or compatible with zero [14]. This can confirm the expectation that the sea quark contribution correlates with the gluon contribution.

In the present approach the quark effective mass ratio $\tilde{\mu}$ and the gluon AM contribution J^g are free, phenomenological parameters constrained by the relation (16). At the same time corresponding scale-dependent parameters, for example the quark effective masses, are (at least in principle) calculable in QCD [28]. However, due to nonperturbative aspect of related task, the real calculation can be extremely difficult. That is why our approach based on the covariant quark-parton model can be a useful supplement to the exact but more complicated theory of the nucleon spin structure based on pure QCD.

To conclude, interpretation of the available sets of experimental data in framework of the covariant approach suggests an important role of the quark OAM for the creation of the proton spin on the scale Q^2 defined by the data. The data of experiments COMPASS and HERMES on the quark and gluon contribution to the proton spin are fully compatible with our approach. At the same time, a limited positive gluon contribution does not contradict the covariant approach. However, the precise data on J^g are still missing, so the existing experimental data do not disprove the hypothesis $J^g \approx 0$ based on the analogy with AM of virtual photons given by Eq. (8).

Acknowledgments

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Appendix A: Proof of the relation (8)

The current (5) generates the electric and magnetic field

$$\mathbf{E}(\mathbf{r}) = \int I_0(\mathbf{r}')\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^{3/2}}d^3\mathbf{r}', \tag{A1}$$

$$\mathbf{H}(\mathbf{r}) = \int \mathbf{I}(\mathbf{r}')\times\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^{3/2}}d^3\mathbf{r}'. \tag{A2}$$

If we define

$$\mathbf{W}^X(\mathbf{r}) = \int \frac{h_X(r')\rho_{j,j_z}(\cos\theta')\,\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^{3/2}}d^3\mathbf{r}', \tag{A3}$$
$$S(\mathbf{r}) = \int \frac{h_I(r')\rho_{j,j_z}(\cos\theta')}{|\mathbf{r}-\mathbf{r}'|^{3/2}}d^3\mathbf{r}',$$

where $X = I, II$, then with the use of (5),(6) we get:

$$\mathbf{E}(\mathbf{r}) = -\mathbf{W}^I(\mathbf{r}) + S(\mathbf{r})\mathbf{r}, \qquad \mathbf{H}(\mathbf{r}) = \mathbf{W}^{II}(\mathbf{r})\times\mathbf{r}. \tag{A4}$$

In terms of spherical coordinates

$$r_1 = r\sin\theta\cos\varphi,\;\; r_2 = r\sin\theta\sin\varphi,\;\; r_3 = r\cos\theta \tag{A5}$$

we have

$$S(\mathbf{r}) = \int \frac{h_I(r')\rho_{j,j_z}(\cos\theta')}{\left(r^2 + r'^2 - 2rr'\left(\sin\theta\sin\theta'\cos(\varphi-\varphi') + \cos\theta\cos\theta'\right)\right)^{3/2}} r'^2 \sin\theta' d\varphi' d\theta' dr', \tag{A6}$$

$$\mathbf{W}^X(\mathbf{r}) = \int \frac{h_X(r')\rho_{j,j_z}(\cos\theta')\,\mathbf{r}'}{\left(r^2 + r'^2 - 2rr'\left(\sin\theta\sin\theta'\cos(\varphi-\varphi') + \cos\theta\cos\theta'\right)\right)^{3/2}} r'^2 \sin\theta' d\varphi' d\theta' dr'. \tag{A7}$$

Obviously $\mathbf{S}(\mathbf{r})$ does not depend on φ so we have

$$S(\mathbf{r}) = S(r,\theta). \tag{A8}$$

In the second integral, after substitution $\psi = \varphi' - \varphi$ we replace correspondingly in $\mathbf{r}'$:

$$x' = r'\sin\theta'\cos\varphi' \rightarrow r'\sin\theta'\left(\cos\psi\cos\varphi - \sin\psi\sin\varphi\right), \tag{A9}$$

$$y' = r'\sin\theta'\sin\varphi' \rightarrow r'\sin\theta'\left(\cos\psi\sin\varphi + \sin\psi\cos\varphi\right) \tag{A10}$$

and instead of (A7) we obtain

$$W_1^X(\mathbf{r}) = \int \frac{h_X(r')\rho_{j,j_z}(\cos\theta')\,r'\sin\theta'\left(\cos\psi\cos\varphi - \sin\psi\sin\varphi\right)}{\left(r^2 + r'^2 - 2rr'\left(\sin\theta\sin\theta'\cos\psi + \cos\theta\cos\theta'\right)\right)^{3/2}} r'^2 \sin\theta' d\psi d\theta' dr', \tag{A11}$$

$$W_2^X(\mathbf{r}) = \int \frac{h_X(r')\rho_{j,j_z}(\cos\theta')\,r'\sin\theta'\left(\cos\psi\sin\varphi + \sin\psi\cos\varphi\right)}{\left(r^2 + r'^2 - 2rr'\left(\sin\theta\sin\theta'\cos\psi + \cos\theta\cos\theta'\right)\right)^{3/2}} r'^2 \sin\theta' d\psi d\theta' dr', \tag{A12}$$

$$W_3^X(\mathbf{r}) = \int \frac{h_X(r')\rho_{j,j_z}(\cos\theta')\,r'\cos\theta'}{\left(r^2 + r'^2 - 2rr'\left(\sin\theta\sin\theta'\cos\psi + \cos\theta\cos\theta'\right)\right)^{3/2}} r'^2 \sin\theta' d\psi d\theta' dr'. \tag{A13}$$

Since in general

$$\int_{-\pi}^{\pi} f_{even}(\psi)\sin\psi d\psi = 0, \tag{A14}$$

where $f_{even}(\psi) = f_{even}(-\psi)$, then the second term in (A11),(A12) vanishes and the expressions are simplified as

$$W_1^X(\mathbf{r}) = W^X(r,\theta)r_1, \qquad W_2^X(\mathbf{r}) = W^X(r,\theta)r_2, \qquad W_3^X(\mathbf{r}) = W_3^X(r,\theta), \tag{A15}$$

where

$$W^X(r,\theta) = \frac{1}{r\sin\theta}\int \frac{h_X(r')\rho_{j,j_z}(\cos\theta')\,r'\sin\theta'\cos\psi}{\left(r^2 + r'^2 - 2rr'\left(\sin\theta\sin\theta'\cos\psi + \cos\theta\cos\theta'\right)\right)^{3/2}} r'^2 \sin\theta' d\psi d\theta' dr', \tag{A16}$$

$$W_3^X(r,\theta) = \int \frac{h_X(r')\rho_{j,j_z}(\cos\theta')\,r'\cos\theta'}{\left(r^2 + r'^2 - 2rr'\left(\sin\theta\sin\theta'\cos\psi + \cos\theta\cos\theta'\right)\right)^{3/2}} r'^2 \sin\theta' d\psi d\theta' dr'. \tag{A17}$$

After inserting from (A4) into (7) we integrate the AM density

$$\mathbf{j}^\gamma = \mathbf{r} \times \left(\left(-\mathbf{W}^I(\mathbf{r}) + S(\mathbf{r})\mathbf{r}\right) \times \left(\mathbf{W}^{II}(\mathbf{r}) \times \mathbf{r}\right)\right), \tag{A18}$$

which with the use of (A8), (A15) gives

$$\mathbf{j}^\gamma = \left\{\begin{array}{c} \left(W^I W^{II}\left(r_1^2 r_2 r_3 + r_2^3 r_3\right) + W^{II}W_3^I r_2 r_3^2 - W^I W_3^{II}\left(r_1^2 r_2 + r_2^3\right) - W_3^I W_3^{II} r_2 r_3\right), \\ \left(-W^I W^{II}\left(r_1 r_2^2 r_3 + r_1^3 r_3\right) - W^{II}W_3^I r_1 r_3^2 + W^I W_3^{II}\left(r_1 r_2^2 + r_1^3\right) + W_3^I W_3^{II} r_1 r_3\right), \\ 0 \end{array}\right\} \tag{A19}$$

$$+ Sr^2\left\{\begin{array}{c} \left(-W^{II} r_2 r_3 + W_3^{II} r_2\right), \\ \left(W^{II} r_1 r_3 - W_3^{II} r_1\right), \\ 0 \end{array}\right\}.$$

These terms depend on φ only via coordinates r_1 and r_2 (A5). Since each term involves just one odd power of $r_1 \sim \cos\varphi$ or $r_2 \sim \sin\varphi$, the corresponding integral satisfies (8).

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